

**A Consistent Theory of Cross Section Mechanics for the  
Application in Material Nonlinear Beam Elements**

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**ABSTRACT**

Nonlinear beam elements used in structural analysis are mainly based on the classical bending theory which describes only the bending effect with accuracy. Shear and torsion can thus only be analysed approximately, in particular when nonlinear material behaviour, e.g. that of reinforced concrete, is involved. However, the combination of the classical bending theory with the theory of plane stress enables all required effects to be calculated with the necessary accuracy.

For a general case of 6 internal forces ( $N, V_y, V_z, T_x, M_y, M_z$ ) and 5 external forces ( $p_x, p_y, p_z, m_x, p_x$ ) 11 corresponding strains and distortion quantities, including their first and second derivatives ( $\varepsilon_x, \kappa_y, \kappa_z, \dots, \vartheta', \vartheta''$ ), are obtained. In addition to these global forces and strains, 3 forces in the local plane ( $n_t, t, n_q$ ) and the corresponding local strains ( $\varepsilon_t, \gamma, \varepsilon_q$ ) have to be considered as well. These unknown strains are used directly in the calculation as degrees of freedom. For a nonlinear material the solution is performed by iteration.

The theory is applicable for every engineering material. A comparison between numerical results and tests shows a very satisfactory agreement even in cases with high shear forces in reinforced and prestressed concrete beams. The theory has already been applied in difficult cases in the engineering practice.

**MOTIVATION**

For the time being the classical bending theory has been applied in nonlinear structural analysis using beam elements (e.g. Eibl and Retzepis 1990). Unfortunately, using this well established theory, shear and torsion effects can only be analysed approximately which in specific cases is not sufficient for design purposes. This is best illustrated by studying the different design codes for reinforced and prestressed concrete. Each code includes a different calculation model for shear and torsion. In addition, these models only satisfy the ultimate limit state but cannot be directly applied for serviceability.

However, the combination of the classical bending theory on the cross section (x-face) with the theory of plane stress applied on each element of the cross section (on the local l-p-plane, see Fig. 1) ensures that all required effects can be calculated with the necessary accuracy for both serviceability and ultimate limit state.

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Using this method, a cross section of any shape can be analysed and different material properties of the cross section can be considered.

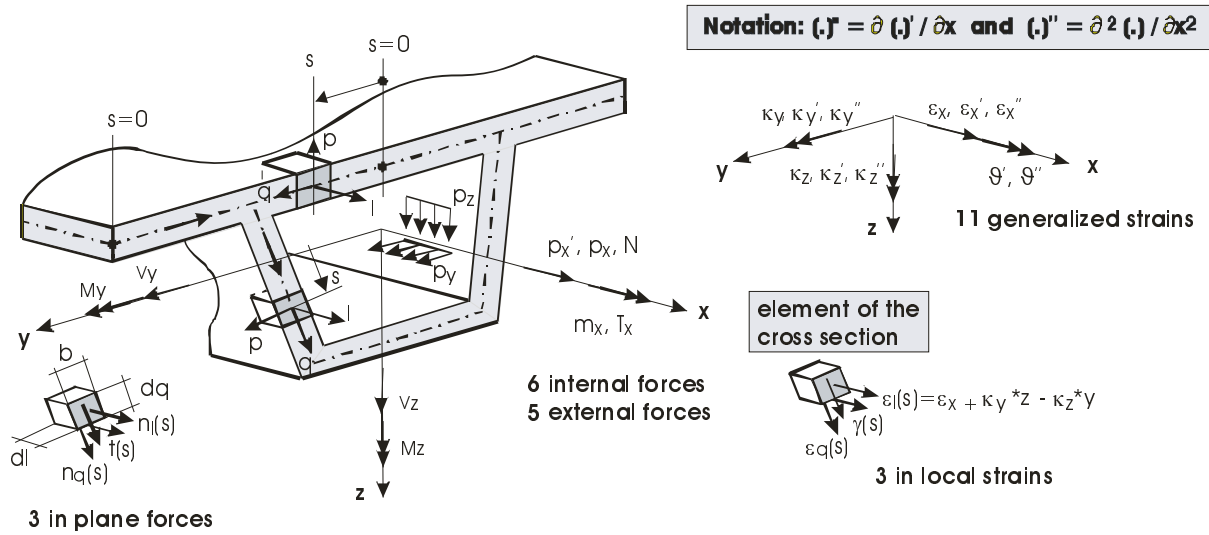


Fig. 1. Force and strain notation at a cross section

Fig. 1 shows a typical cross section of a bridge with the force and strain notation which will be used. The cross section is idealized by its middle line. In general, 6 internal global forces ( $N$ ,  $V_y$ ,  $V_z$ ,  $T_x$ ,  $M_y$ ,  $M_z$ ) and 5 external global forces ( $p_x$ ,  $p_y$ ,  $p_z$ ,  $m_x$ ,  $p_x'$ ) act on the cross section. The 11 corresponding global strains and distortion values including their first and second derivatives ( $\epsilon_x$ ,  $\kappa_y$ ,  $\kappa_z$ , ...,  $\theta'$ ,  $\theta''$ ) are also displayed in Fig 1. In addition to the strains already mentioned, the local forces and strains acting on each element cut out of the cross section are also obtained; these values are the 3 forces in the local plane ( $n_l$ ,  $t$ ,  $n_q$ ) and the corresponding local strains ( $\epsilon_l$ ,  $\gamma$ ,  $\epsilon_q$ ). All these unknown strains are directly used in the calculation as degrees of freedom. It is obvious that for a nonlinear material the solution can only be performed by iteration. Because of the higher global strain derivatives and the local strains which are also involved as degrees of freedom, a five level iteration is applied as will be shown later.

## ASSUMPTIONS

The strict assumptions of the classical bending theory (e.g. Roik 1972) can be extended to the following more general assumptions:

1. Uniform warping over the cross section
2. Warping effects due to line load can be neglected
3. Geometry of the cross section remains unchanged

With the above assumptions, the geometric and static equations can be derived in a direct way. The application of the theory to any nonlinear material doesn't involve any further complication as will be shown below. Moreover, any nonlinear material describable as a function of strain can be used.

## EQUILIBRIUM CONDITIONS

From the well known equilibrium relationship applied on an element cut out of the cross section (Fig. 1)

$$\frac{\partial t}{\partial q} + \frac{\partial n_l}{\partial l} = 0 \quad (1.1)$$

$$\frac{\partial n_q}{\partial q} + \frac{\partial t}{\partial l} = 0 \quad (1.2)$$

the following expressions can be derived:

$$t(s) = - \int_{s=0}^s \frac{\partial n_l}{\partial l} dq + C_0 \quad (2.1)$$

$$n_q(s) = \int_{s=0}^s \left( \int_{s=0}^q \frac{\partial^2 n_l}{\partial l^2} dq + C_1 \right) dq + C_2 \quad (2.2)$$

The expression Eq.(2.2) results from the partial differentiation of Eq.(1.2) by  $\partial l$  and using Eq.(1.1). These expressions allow the evaluation of the forces acting on the q-face using the forces on the l-face of the cross section (Fig. 1). The constants  $C_0$ ,  $C_1$  and  $C_2$  can be evaluated, as will be shown later, using deformation criteria.

In addition, for width cross section elements the forces perpendicular to the local l-q-plane in the direction p have to be considered (Fig. 2). These are the shear forces  $t_{lp}$  and  $t_{qp}$ . For further calculation these forces are transformed into the global coordinate system expressed as  $t_y$ ,  $t_z$ ,  $v_y$  and  $v_z$ .

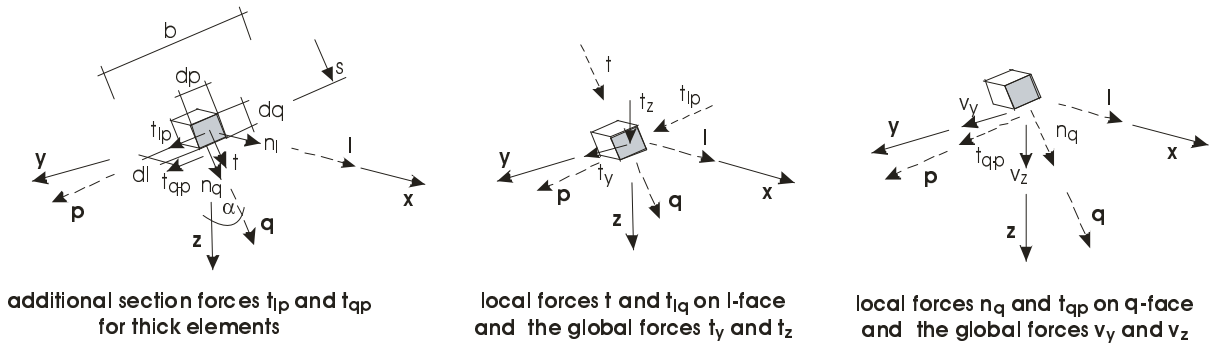


Fig. 2. Section forces at the element

The well known global equilibrium relationships of the beam theory are:

$$\begin{aligned} \frac{\partial N}{\partial x} &= -p_x, & \frac{\partial V_y}{\partial x} &= -p_y, & \frac{\partial V_z}{\partial x} &= -p_z \\ \frac{\partial T_x}{\partial x} &= -m_x, & \frac{\partial M_y}{\partial x} &= V_z, & \frac{\partial M_z}{\partial x} &= -V_y \\ \frac{\partial^2 N}{\partial x^2} &= 0, & \frac{\partial^2 M_y}{\partial x^2} &= -p_z, & \frac{\partial^2 M_z}{\partial x^2} &= p_y \end{aligned} \quad (3)$$

Using the above relationships the (integrated) shear forces  $V_z(s)$  and  $V_y(s)$  can be calculated from the starting points of the cross section  $s = 0$  to the present location  $s$  (Hartung and Krebs 2004):

$$\begin{aligned} V_z(s) &= -z(s) * N_x'(s) + M_y'(s) - z(s) * \sum_0^s p_{x,k} + \sum_0^s z_k * p_{x,k} \\ V_y(s) &= -y(s) * N_x'(s) - M_z'(s) - y(s) * \sum_0^s p_{x,k} + \sum_0^s y_k * p_{x,k} \end{aligned} \quad (4)$$

Hereby, the values  $N'_x, M'_y, M'_z$  are the first derivatives of the force ( $N'_x = \partial N_x / \partial x$ ) and moments. For each closed part of the cross section, a section cut is introduced so that a statically determined system is built. The forces at the section cut are calculated using continuity conditions. Using these expressions the shear forces  $t_z(s_m)$  and  $t_y(s_m)$  on the element can easily be evaluated:

$$\begin{aligned} t_z(s_m) &= \frac{\Delta V_z(s_m)}{\Delta s} \text{ with } \Delta V_z(s_m) = V_z(s) - V_z(s-1) \\ t_y(s_m) &= \frac{\Delta V_y(s_m)}{\Delta s} \text{ with } \Delta V_y(s_m) = V_y(s) - V_y(s-1) \end{aligned} \quad (5)$$

From these shear forces in the global coordinate system  $t_z(s_m)$  and  $t_y(s_m)$ , the forces  $t(s_m)$  and  $t_{ip}(s_m)$  in the local coordinate system are determined.

In a similar way the forces  $v_y(s)$  and  $v_z(s)$  are calculated:

$$\begin{aligned} v_z(s) &= z(s) * N''_x(s) - M''_y(s) + z(s) * \sum_0^s p_{x,k} - \sum_0^s z_k * p_{x,k} - \sum_0^s p_{z,k} \\ v_y(s) &= y(s) * N''_x(s) + M''_z(s) + y(s) * \sum_0^s p_{x,k} - \sum_0^s y_k * p_{x,k} - \sum_0^s p_{y,k} \end{aligned} \quad (6)$$

The values  $N''_x, M''_y, M''_z$  are the second derivatives of the force and moments. From these values the forces  $v_z(s_m)$  and  $v_y(s_m)$  on the element can be evaluated:

$$\begin{aligned} v_z(s_m) &= \frac{1}{2} * (v_z(s) + v_z(s-1)) \\ v_y(s_m) &= \frac{1}{2} * (v_y(s) + v_y(s-1)) \end{aligned} \quad (7)$$

The normal force  $n_q(s_m)$  and the shear force  $t_{qp}(s_m)$  on the q-face are determined as demonstrated over a transformation of the above global forces  $v_z(s_m)$  and  $v_y(s_m)$  into the local system.

At the starting points  $s = 0$  of the idealised cross section the force values can be determined using deformation boundary conditions which will be shown below.

## GEOMETRIC CONDITIONS

The state of strain on the x-face is described using not only the strain values ( $\epsilon_x, \kappa_y, \kappa_z$ ) but also their first and second derivatives ( $\epsilon'_x, \epsilon''_x, \kappa'_y, \kappa''_y, \kappa'_z, \kappa''_z$ ) (Fig. 1). For each point of the cross section, the local strain  $\epsilon_{l,i}$  can be determined in the usual way  $\epsilon_{l,i} = \epsilon_x + z_i * \kappa_y - y_i * \kappa_z$ . The normal strain  $\epsilon_{q,i}$  on the q-face and the shear strain  $\gamma_i$  are assumed to be constant in the x-direction of each element of the cross section so that their derivatives are zero.

The consideration of the torsion effects requires the introduction of the first and second derivatives of the angle of twist  $\vartheta', \vartheta''$ . At each section cut, which was introduced earlier for a closed part of a cross section, the four displacement values  $\Delta u, \Delta v, \Delta w, \Delta \varphi$  are introduced and they have to fulfil the conditions of a closed cross section (Fig. 3):

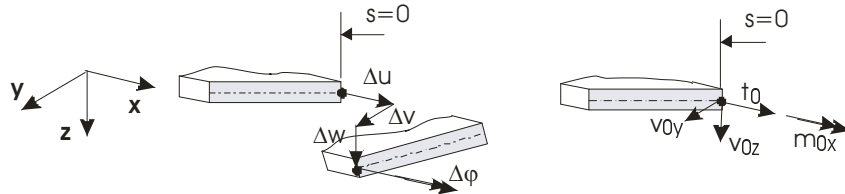


Fig. 3. Distortion at a section cut

$$\Delta u = 0 \text{ and} \quad (8)$$

$$\Delta v = 0, \Delta w = 0, \Delta \varphi = 0$$

In this calculation, the forces  $t_0$ ,  $v_{0y}$ ,  $v_{0z}$  and the moment  $m_{0x}$  are involved. They can be determined using the classical method of consistent displacements.

### CONSTITUTIVE EQUATIONS

The applied constitutive equations allow the direct evaluation of the stress using the corresponding strains, with both stress and strains expressed in their principal directions. The stress and strains are assumed to be coaxial. Fig. 4 shows the stress-strain diagrams for reinforced steel and concrete which are used for a reinforced concrete cross section. The modelling of the tension stiffening effect (TS) has been carried out in a simple way by modifying the stress-strain diagram of concrete in the tension region.

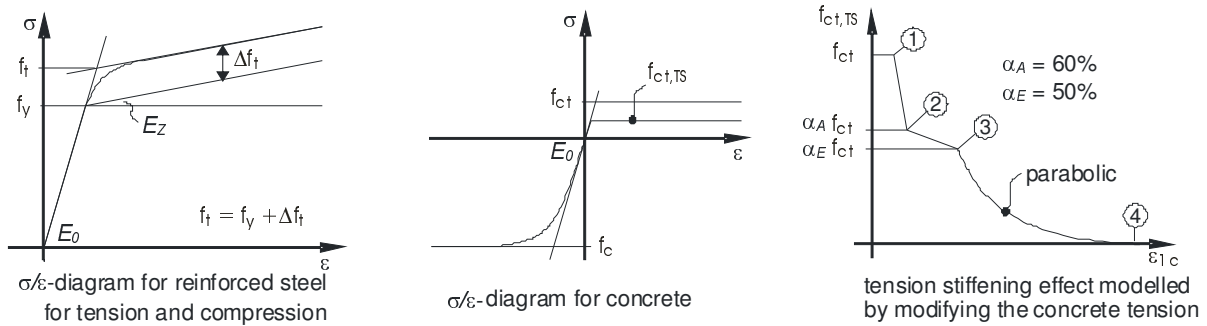


Fig. 4. Stress-strain diagrams for steel and concrete

The above stress-strain diagrams correspond to the following relationship

$$\sigma_1 = f_1(\varepsilon_1, \varepsilon_2) \quad (9)$$

$$\sigma_2 = f_2(\varepsilon_1, \varepsilon_2)$$

which is valid for an elastic material. For structural steel an elasto-plastic model with flow rule is used. The procedure can be extended to more sophisticated constitutive equations.

### SOLUTION

Considering the equilibrium and the geometric conditions mentioned above, the following scheme can be drawn in which the force and the corresponding strains or distortion values are expressed. For calculation reasons a differentiation between three levels is applied:

*Level 1 (cross section as a whole part)*

Level	Force Values	Strain or Distortion Values
1.1	$N, M_y, M_z$	$\varepsilon_x, \kappa_y, \kappa_z$
1.2	$N' = -p_x, M_y' = V_z, M_z' = -V_y$	$\varepsilon_x', \kappa_y', \kappa_z'$
1.3	$N'' = 0, M_y'' = -p_z, M_z'' = p_y$	$\varepsilon_x'', \kappa_y'', \kappa_z''$

Level 2 (closed part(s) of the cross section)

Force Values	Strain or Distortion Values
$\bar{T}_x, t_0$	$\mathcal{G}', \Delta u = 0$
$\bar{T}_x' = -m_x, t_0'$	$\mathcal{G}'', \Delta u' = 0$
$v_{0y}, v_{0z}, m_{0x}$	$\Delta v = 0, \Delta w = 0, \Delta \varphi = 0$

Level 3 (for each element of the cross section)

Force Values	Strain or Distortion Values
$t, n_q$	$\gamma, \varepsilon_q$

It is obvious that there are as many equations as unknown values. From this point of view a solution is possible. In the general case of a nonlinear material an iterative solution is evident. The unknown forces, strains or distortion values can be directly used as degrees of freedom (DOF). Level 3 is the inner loop, followed by level 2 and ending with the outer loop of level 1. Taking also the derivatives into account (1<sup>st</sup> and 2<sup>nd</sup> derivatives in level 1), a five level iteration is used.

Substituting the constitutive equations in the equilibrium conditions, the following equations are derived for the above mentioned levels.

For Level 1:

$$\begin{aligned}
 & \begin{bmatrix} K_{\varepsilon_x} & K_{\varepsilon_y \kappa_y} & K_{\varepsilon_x \kappa_z} \\ K_{\kappa_y \varepsilon_x} & K_{\kappa_y} & K_{\kappa_y \kappa_z} \\ K_{\kappa_z \varepsilon_x} & K_{\kappa_z \kappa_y} & K_{\kappa_z} \end{bmatrix} * \begin{bmatrix} \varepsilon_x \\ \kappa_y \\ \kappa_z \end{bmatrix} = \begin{bmatrix} N \\ M_y \\ M_z \end{bmatrix} - \begin{bmatrix} R_N \\ R_{M_y} \\ R_{M_z} \end{bmatrix} \\
 & \begin{bmatrix} K_{\varepsilon_x'} & K_{\varepsilon_x' \kappa_y'} & K_{\varepsilon_x' \kappa_z'} \\ K_{\kappa_y' \varepsilon_x'} & K_{\kappa_y'} & K_{\kappa_y' \kappa_z'} \\ K_{\kappa_z' \varepsilon_x'} & K_{\kappa_z' \kappa_y'} & K_{\kappa_z'} \end{bmatrix} * \begin{bmatrix} \varepsilon_x' \\ \kappa_y' \\ \kappa_z' \end{bmatrix} = \begin{bmatrix} -p_x \\ V_z \\ -V_y \end{bmatrix} - \begin{bmatrix} R_{p_x} \\ R_{V_z} \\ R_{V_y} \end{bmatrix} \\
 & \begin{bmatrix} K_{\varepsilon_x''} & K_{\varepsilon_x'' \kappa_y''} & K_{\varepsilon_x'' \kappa_z''} \\ K_{\kappa_y'' \varepsilon_x''} & K_{\kappa_y''} & K_{\kappa_y'' \kappa_z''} \\ K_{\kappa_z'' \varepsilon_x''} & K_{\kappa_z'' \kappa_y''} & K_{\kappa_z''} \end{bmatrix} * \begin{bmatrix} \varepsilon_x'' \\ \kappa_y'' \\ \kappa_z'' \end{bmatrix} = \begin{bmatrix} 0 \\ -p_z \\ p_y \end{bmatrix} - \begin{bmatrix} R_0 \\ R_{p_z} \\ R_{p_y} \end{bmatrix}
 \end{aligned} \tag{10}$$

$$\text{with } K_{\varepsilon_x} = \sum_{i=1}^n A_{ci} * (E_{s1c} * \cos^4 \varphi_c + E_{s2c} * \sin^4 \varphi_c)$$

$$\text{and } R_N = \sum_{i=1}^n A_{ci} * \left( (E_{s1c} * \cos^3 \varphi_c * \sin \varphi_c - E_{s2c} * \cos \varphi_c * \sin^3 \varphi_c) * \gamma_i + (E_{s1c} + E_{s2c}) * \cos^2 \varphi_c * \sin^2 \varphi_c * \varepsilon_{qi} \right) \text{ for concrete}$$

whereby  $A_{ci}$  is the area of a cross section element,  $E_{s1c}$  and  $E_{s2c}$  are the secant modulus of elasticity of concrete in the principal strain directions,  $\varphi_c$  is the angle between the 1<sup>st</sup> principal strain direction and the x-axis and  $\gamma_i$  and  $\varepsilon_{qi}$  are the two strains at the element. For a reinforced and prestressed cross section additional terms for the reinforcement are involved. Obviously the summation has to be performed over the cross section. Much more complicated terms are involved for the 1<sup>st</sup> and 2<sup>nd</sup> derivatives of the strains.

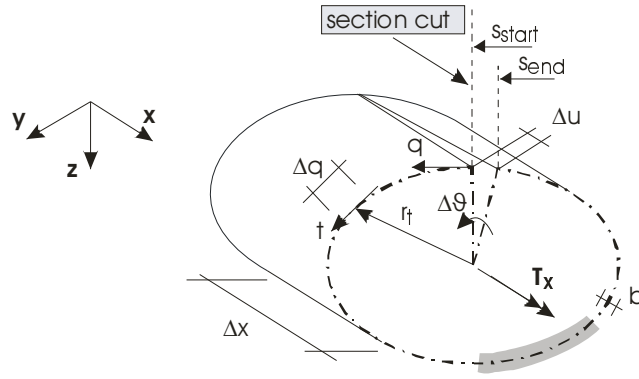


Fig. 5. Torsion on a closed part of the cross section

For Level 2:

$$\begin{aligned}
 \begin{bmatrix} 0 & K_{g^+ t_0} \\ K_{t_0 g^+} & K_{t_0} \end{bmatrix} * \begin{bmatrix} g^+ \\ t_0 \end{bmatrix} &= \begin{bmatrix} T_x \\ \Delta u = 0 \end{bmatrix} - \begin{bmatrix} R_{T_x} \\ R_{\Delta u = 0} \end{bmatrix} \\
 \begin{bmatrix} 0 & K_{g^- t_0} \\ K_{t_0 g^-} & K_{t_0} \end{bmatrix} * \begin{bmatrix} g^- \\ t_0 \end{bmatrix} &= \begin{bmatrix} m_x \\ \Delta u = 0 \end{bmatrix} - \begin{bmatrix} R_{m_x} \\ R_{\Delta u = 0} \end{bmatrix} \\
 \begin{bmatrix} K_{v_{0z}} & \cdot & \cdot \\ \cdot & \dots & \cdot \\ \cdot & \cdot & K_{m_{0x}} \end{bmatrix} * \begin{bmatrix} v_{0z} \\ v_{0y} \\ m_{0x} \end{bmatrix} &= \begin{bmatrix} \Delta w \\ \Delta v \\ \Delta \varphi \end{bmatrix} = 0
 \end{aligned} \tag{11}$$

The first two expressions result directly from the classical St. Venant torsion theory (e.g. Peterson 1988) being applied on each closed part of the cross section (Fig. 5)

$$T_x = \oint t * r_t * dq \quad \text{and} \quad \Delta u = \oint \gamma * dq - \frac{\partial g}{\partial x} * \oint r_t * dq \tag{11.1}$$

and the last expressions are derived using the classical method of consistent displacements at the section cut (conditions of continuity).

For Level 3:

$$\begin{bmatrix} K_\gamma & K_{\gamma \varepsilon_q} \\ K_{\varepsilon_q \gamma} & K_{\varepsilon_q} \end{bmatrix} * \begin{bmatrix} \gamma \\ \varepsilon_q \end{bmatrix} = \begin{bmatrix} t \\ n_q \end{bmatrix} \tag{12}$$

where  $K_\gamma = A_{ci} * \left[ (E_{t1c} + E_{t2c}) * \cos^2 \varphi_c * \sin^2 \varphi_c + \frac{1}{2} * \frac{\sigma_{1c} - \sigma_{2c}}{\varepsilon_{1c} - \varepsilon_{2c}} * \cos^2 2\varphi_c \right]$  for concrete.

$A_{ci}$  being the area of a cross section element,  $E_{t1c}$  and  $E_{t2c}$  the (tangent) modulus of elasticity of concrete in the principal strain directions,  $\varphi_c$  the angle between the 1<sup>st</sup> principal strain direction and the x-axis and  $\sigma_{1c}$ ,  $\sigma_{2c}$ ,  $\varepsilon_{1c}$ ,  $\varepsilon_{2c}$  the principal stresses and strains at the element. As was mentioned earlier, additional terms for the reinforcement are involved for reinforced and prestressed cross sections.

The elements  $K_{ij}$  of the stiffness matrixes can be calculated analytically. Examples have been given for the easiest elements of the matrix  $K_{\varepsilon_x}$ ,  $K_\gamma$ .  $R_N$  is an element of the load vectors. However, the calculation of the other terms is cumbersome (Hartung 1985). An iterative solution procedure using the Newton-Raphson method combined with an underrelaxation (using a variable relaxation

parameter  $\beta$ ) is applied. The iteration parameters (DOF) are the unknown above mentioned strain and distortion values.

The algorithm has been implemented in a developed program code called KuK-QuMe. Using this program the state of strain can be determined for the general case of 6 internal forces ( $N, V_y, V_z, T_x, M_y, M_z$ ) and 5 external forces ( $p_x, p_y, p_z, m_x, p_x'$ ). As was mentioned at the beginning, any arbitrary cross section can be handled for steel, reinforced or prestressed concrete as well as composite material (steel-concrete).

## VERIFICATION

Intensive verification tests have been carried out. At the beginning the focus was on the problems of the theory of linear elasticity, especially in cases where an analytical solution exists. Then complicated cross sections were analysed, like that of Fig. 6. The steel box girder consists of three compartments and has been numerically analysed using finite elements (Wimmer 2000). Fig. 6 shows the cross section and the distribution of shear stress as well as warping due to the force  $V_z$ . There is a very good agreement between the numerical results using the developed theory and the published values.

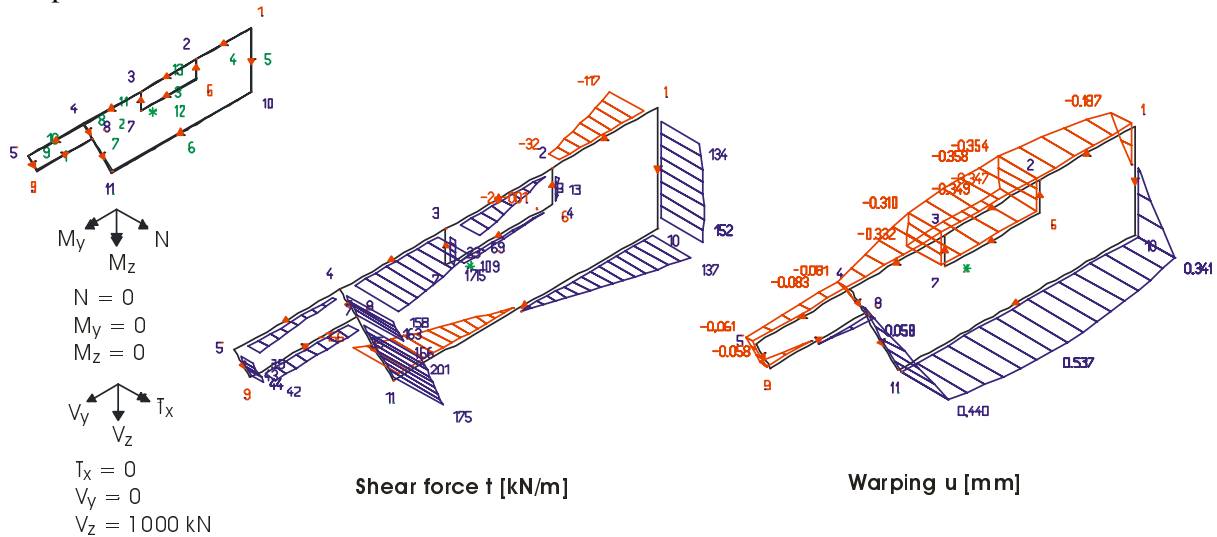


Fig. 6. Cross section, internal forces and warping due to force  $V_z$

The main emphasis of the developed theory however is the application to nonlinear problems. The centre of interest is – as mentioned earlier – the nonlinear behaviour of reinforced as well as prestressed concrete cross section under a combined action of forces and moments. Any type of prestressing, using bonded or unbonded tendons as well as a combination of these two types is applicable with the developed theory. After verification of the theory for load cases involving normal forces and bending moments, the focus was on cases with high shear forces. Many tests which have been carried out in Germany were used for verification.

An additional problem in nonlinear calculations lies in the fact that numerical results similar to those of the experiment can also be obtained by using a wrong combination of material parameters (Eibl et al 1994). This can be explained by the amount of material parameters involved in such cases and can be avoided by verification with different tests. An alternative would be to perform the numerical pre-calculations before starting the test. This has been applied in the case of the two beam tests carried out at the Kaiserslautern University, Germany. At the beginning the material parameters have been evaluated using tests on concrete specimens, e.g. cube tests. Using this theory, the critical load for shear failure has been calculated for the two beams. The first beam was prestressed using internal unbonded tendons, the second was a plain reinforced beam of the same

cross section. Based on the numerical results, the test beams were loaded with an increasing shear force till the point of failure.

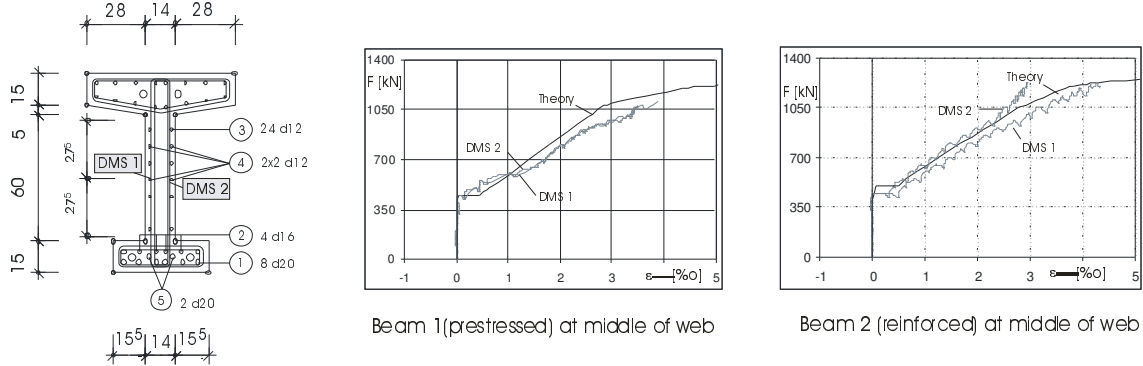


Fig. 7. Cross section and strains in the stirrups of beam 1 and 2

Fig. 7 shows the cross section as well as the strains in the stirrups from the calculation (theory) and the test (DMS 1 and DMS 2). A very satisfactory agreement has been achieved. For further information see Krebs et al (2004).

### APPLICATIONS

The behaviour of reinforced as well prestressed concrete, steel and composite cross section under a combined loading can be illustrated using interaction diagrams. They display the load-bearing capacity of the cross section and consequently such diagrams demonstrate the existing safety factor between the given loading state and the limit state.

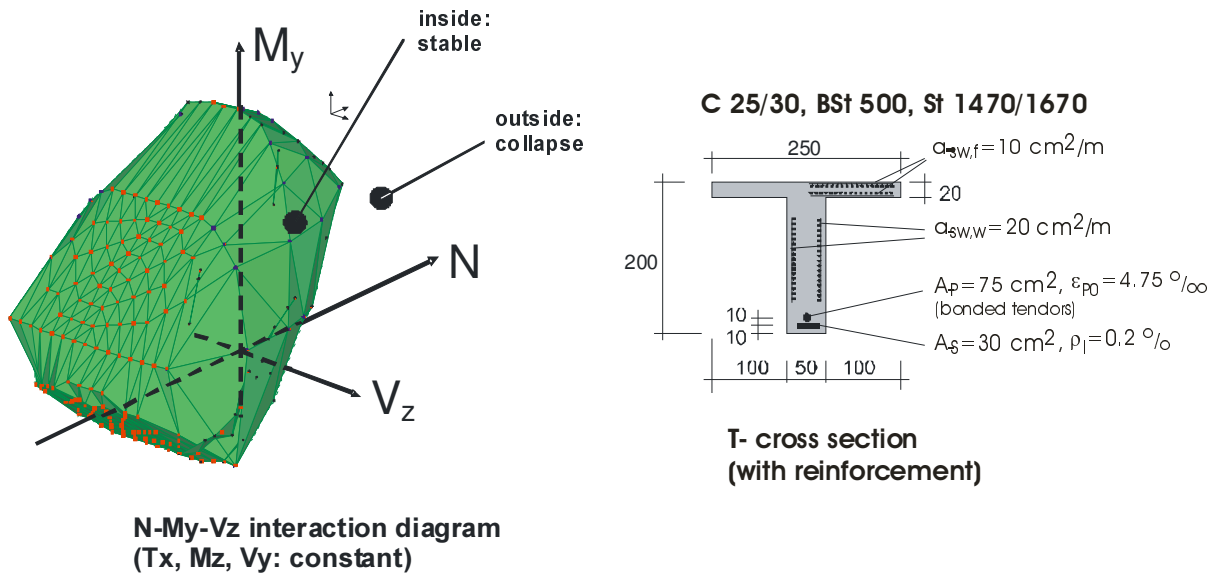


Fig. 8. Interaction diagramm for a prestressed T- cross section

Fig. 8 shows the interaction diagram  $N-M_y-V_z$  of a prestressed T- cross section. The reinforcement consists of mild steel bars and prestressed bonded tendons. The material data are also given. All combinations of the  $N-M_y-V_z$  values lying inside the volume are stable contrary to those outside the volume that indicate the collapse of the cross section.

The theory has already been put into practice for the case of a bridge with a fault during concreting. Applying the standard design method according to the German code DIN 4227, which was valid at the time of the bridge construction, a safety deficit was obtained for this structure (global safety factor  $1.43 < 1.75$ ). However, with the application of the developed theory, the ultimate limit state (global safety factor  $1.76 \approx 1.75$ ) as well as the serviceability was proved, resulting in a moderate rehabilitation of the bridge (Krebs et al 2004). Otherwise, costly strengthening measures for the structure would have needed to be taken.

## CONCLUSION

The present theory, which is a combination of the classical bending theory and the theory of plane stress, allows the consideration of all required effects of bending combined with shear and torsion. The theory is applicable for every arbitrary cross section. Although, at present, a simplified material behaviour of reinforced and prestressed concrete, steel and composite cross sections has been implemented in the program, it has nonetheless proved sufficient for analysis purposes. An extension to more sophisticated material behaviour is possible.

The verification of the theory includes published analytical and numerical results as well as tests on beams carried out in Germany. The verification results show a very satisfactory agreement of the theory with the tests, even in cases involving high shear forces, in reinforced and prestressed concrete beams.

In addition to the above described analysis of individual cross sections, the theory has also been implemented into an existing program for bridge design. Using this program a nonlinear analysis using beam elements can be performed. In this way a realistic response of beam structures can be evaluated and the ultimate as well as the serviceability limit state can be checked.

Applying this theory, the interaction of bending, shear and torsion can be studied which leads to a better understanding of the behaviour of structures. Furthermore, with this knowledge a more effective design of structures is possible. This theory has already been applied on an existing bridge, yielding a more realistic result than the traditional methods which permitted a more moderate rehabilitation of the bridge.

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